DOT INSTITUTE OF PHYSICS

PART-TEST-3

ANSWERS

PHYSICS, CHEMISTRY, MATHEMATICS

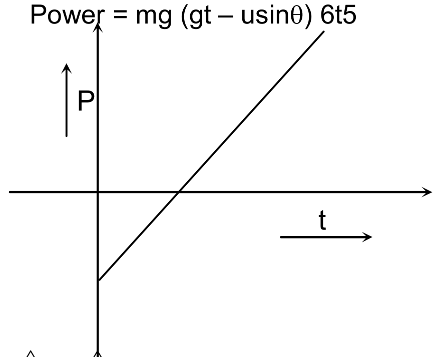
1	D	16	C	31	B	46	A	61	A	76	B
2	D	17	C	32	C	47	B	62	B	77	C
3	A	18	C	33	D	48	D	63	A	78	A
4	A	19	B	34	D	49	A	64	D	79	B
5	A	20	B	35	B	50	A	65	D	80	C
6	A	21	D	36	B	51	A	66	B	81	B
7	A	22	C	37	C	52	A	67	D	82	D
8	D	23	C	38	B	53	A	68	A	83	A
9	B	24	D	39	C	54	D	69	A	84	A
10	C	25	A	40	A	55	B	70	C	85	A
11	B	26	C	41	A	56	D	71	C	86	A
12	C	27	B	42	B	57	C	72	A	87	C
13	A	28	B	43	D	58	B	73	D	88	C
14	A	29	C	44	C	59	D	74	B	89	C
15	C	30	B	45	B	60	C	75	B	90	B

HINTS AND SOLUTIONS

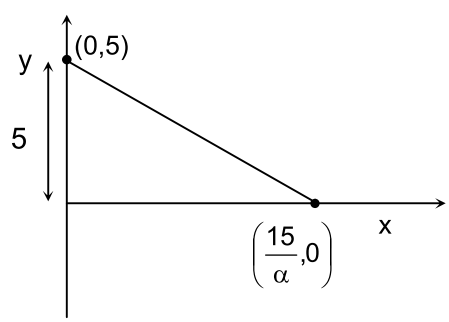
PHYSICS, CHEMISTRY, MATHEMATICS

1. (d) In half of the motion (ascent) the power is negative and half of motion (descent) the power is positive.

Power = mg (gt – usinθ) 6t5



2. (d) $\vec{F} = 20\hat{i} + 15\hat{j}$, 3y = 5 –αx, y = $\frac{-\alpha}{3}x + 5$



∴ graph of equation

∴ $\vec{S} = \frac{15}{\alpha}\hat{i} - 5\hat{j}$

as per question

$$W = \vec{F} \cdot \vec{S} = 0$$

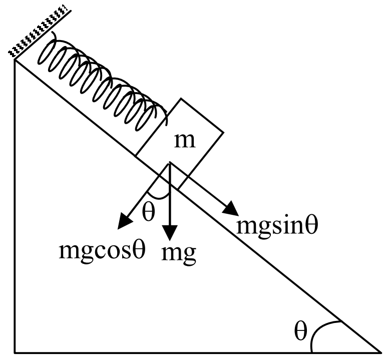
$$W \Rightarrow (20\hat{i} + 15\hat{j}) \cdot \left(\frac{15}{\alpha}\hat{i} - 5\hat{j}\right) = 0$$

$W \Rightarrow 20 \times \frac{15}{\alpha} - 5 \times 15 = 0$

$\frac{20 \times 15}{\alpha} = 15 \times 5$

$\therefore \quad \alpha = 4.$

3. (a)



Let the elongation in spring be 'x'

$\Rightarrow$ Loss of potential energy = work done by gravity = mg sinθ x

$\Rightarrow$ gain in spring potential energy = $\frac{1}{2}kx^2$

$\therefore \quad mg\sin\theta\, x = \frac{1}{2}kx^2$

$\frac{2mg\sin\theta}{k} = x$

4. (a) $\vec{F} = (3t\hat{i} + 5\hat{j})$ $\quad \vec{S} = (2t^2\hat{i} - 5\hat{j})$ m

$W = \vec{F}\cdot\vec{S} = (6t^3 - 25)$

$W_{(t=2)} = (6 \times (2)^3 - 25) = (48 - 25) = 23$ J

5. (a) By work energy theorem.

$mg(h + d) = F \times d$

$\frac{mg(h+d)}{d} = F$

or $\quad F = mg\left(1 + \frac{h}{d}\right)$

6. (a)

A

$F = K_A x$

hence $K_A x = K_B y \Rightarrow$

$E_A = \frac{1}{2} K_A x^2$

$E_A = E$

B

$F = K_B y$

$K_A x = 2K_A y,\ x = 2y$

$E_B = \frac{1}{2} K_B y^2$

$E_B = \frac{1}{2} \times (2K_A)\left(\frac{x}{2}\right)^2$

$= \frac{1}{2} \times 2K_A \times \frac{x^2}{4} = \frac{E}{2}$

7. (a) m = 0.5 kg $\quad$ v = 1.5 m/s

k = 5ON – m

using conservation of energy

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\Rightarrow \frac{m}{k}v^2 = x^2$

$\Rightarrow x = v\sqrt{\frac{m}{k}} = 1.5\sqrt{\frac{0.5}{50}} = 1.5\sqrt{\frac{1\times 5}{500}}$

$\Rightarrow x = 0.15$ m

8. (d) As per question Let the thickness of one plank 'x'

$v^2 - u^2 = 2a\,(2x)$

or $(0)^2 - u^2 = -\,4ax$

or $u^2 = 4ax$ ……………(i)

If speed is doubled.

$(2u)^2 = 2(a)\,(nx)$

$4u^2 = 2a\,nx$ …………….(ii)

using (i) and (ii)

$n = 8$

9. (b) Use the conservation of energy.

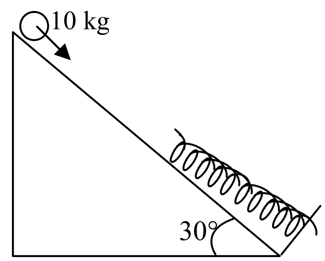
Initial energy = final energy.

10. (c) As per question

$F = 100$ N $\quad x = 1$ m

$F = kx$

$\therefore\ 100 = k(1) \quad$ hence $k = 100$ N/m



Let the distance between spring and Mass be 'x'

$\therefore$ total distance travelled (x + 2) m

using Work Energy theorem

$$mg \sin 30^\circ\,(x + 2) = \frac{1}{2}kx^2$$

$$10 \times 10 \times \frac{1}{2}\,(x + 2) = \frac{1}{2}\times 100\,(x)^2$$

$$(x + 2) = x^2$$

$\Rightarrow\quad x^2 - x - 2 = 0$

$\Rightarrow\quad x^2 - 2x + x - 2 = 0$

$\Rightarrow\quad x(x - 2) + 1\,(x - 2) = 0$

$\Rightarrow\quad (x + 1)\,(x - 2) = 0$

Hence x = 2m

Total distance = 4 m

11. (b) Using the work energy theorem.

12. (c) Work is not done by gravity. The correct statement is work done against gravity.

13. (a) Basic knowledge

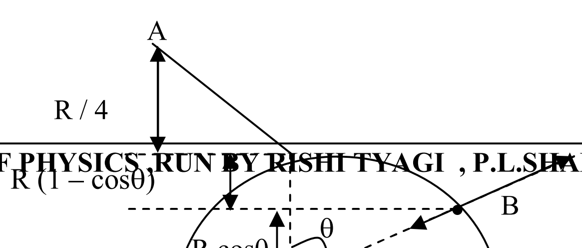
14. (a) Since the velocity of body is decreasing

$\therefore$ net force opposes the motion of body.

15. (c) Since 20% of energy will be lost by 80% of energy it will achieve 80% of height.

$$\frac{80}{100}\times 10 = 8 \text{ m.}$$

16. (c)



from A to B

Total loss in P. E. = gain in K.E.

$$\frac{mgR}{4} + mg\,R\,(1 - \cos\theta) = \frac{1}{2} m\, v^2$$

$$\frac{gR}{4} + gR - g\,R\cos\theta = \frac{1}{2} v^2$$

$$2\left(\frac{5}{4}gR - gR\cos\theta\right) = v^2$$

$$\frac{5}{2}gR - 2gR\cos\theta = v^2 \qquad \text{......(i)}$$

also it loses contact at B (N = O)

$$mg\cos\theta - N = \frac{mv^2}{k}$$

$$mg\cos\theta = \frac{mv^2}{R}$$

$$v^2 = Rg\cos\theta \qquad \text{......(ii)}$$

Putting (ii) in (i)

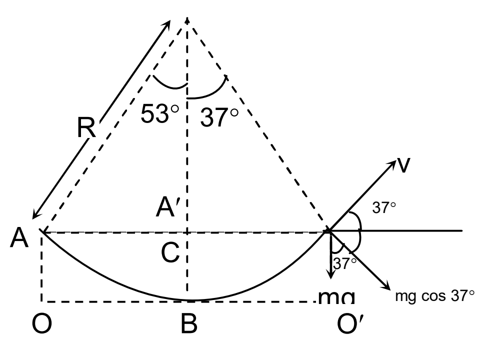
$$\Rightarrow \frac{5}{2}gR - 2g\,R\cos\theta = Rg\cos\theta$$

$$\Rightarrow \frac{5}{2}gR = 3gR\cos\theta$$

$$\Rightarrow \frac{5}{6} = \cos\theta$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{5}{6}\right)$$

17. (c)



if OBO′ is reference point, then

$$OA = A'B = R - R\cos 53^\circ = R - R \times \frac{3}{5}$$

$$= \frac{2R}{5}$$

$$O'C = BC' = R - R\cos 37^\circ = R - R \times \frac{4}{5} = \frac{R}{5}$$

Loss of P.E. going from A to C = gain in K.E.

$$mg\left(\frac{2R}{5}-\frac{R}{5}\right) = \frac{1}{2}mv^2$$

$$\Rightarrow g\left(\frac{R}{5}\right) = \frac{1}{2}v^2$$

$\Rightarrow 4R = v^2$

At point 'c'

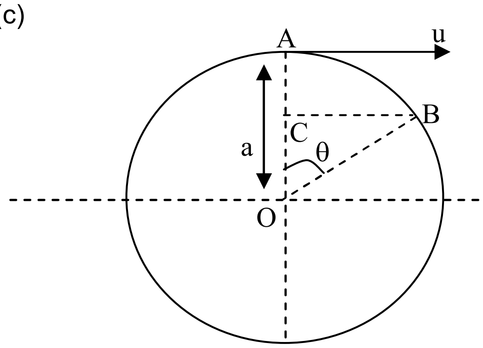
$$\frac{mv^2}{r} = mg\cos 37°$$

$$r = \frac{v^2}{g\cos 37°}$$

$$r = \frac{4R}{g\times\frac{4}{5}}$$

$$\therefore r = \frac{R}{2}$$

18. (c)



$$\Rightarrow AC = \frac{a}{4}$$

$OC = OA – AC$

$$= a - \frac{a}{4}$$

$$OC = \frac{3a}{4}$$

also $OB = a$

Loss of P.E. = gain in K.E. s

$$mga - mg\times\frac{3}{4}a = \frac{1}{2}m\,x^2 - \frac{1}{2}mu^2$$

$$\frac{mga}{4} = \frac{1}{2}m\,(x^2 - u^2)$$

$$\frac{ga}{2} = (x^2 - u^2)$$

for leaving sphere.

$$mg\cos\theta = \frac{mx^2}{a}$$

$x^2 = ga\cos\theta$

also from figure $\cos\theta = \frac{3a}{4\times a} = \frac{3}{4}$

$$\therefore x^2 = \frac{3ga}{4}$$

Hence

$$\frac{ga}{2} = \frac{3ga}{4} - u^2$$

$$u^2 = \frac{3ga}{4} - \frac{ga}{2}$$

$$u^2 = \frac{3ga - 2ga}{4} = \frac{ga}{4}$$

$$u = \frac{\sqrt{ga}}{2}$$

19. (b) Mass of particle = m
radius of circle = r

Centripetal acceleration = $\frac{4}{r^2}$

as per given information

$$\frac{v^2}{r} = \frac{4}{r^2}$$

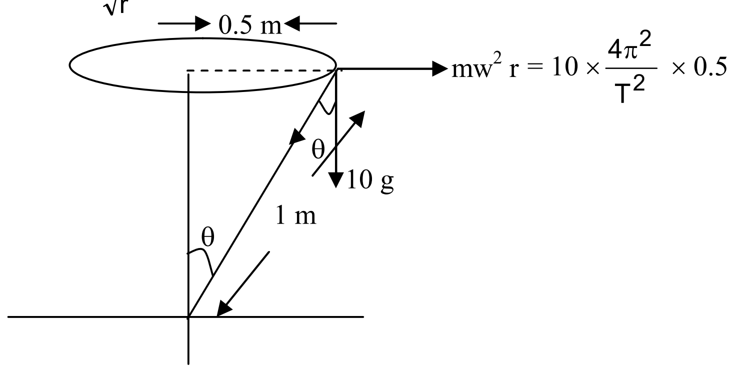
$$v^2 = \frac{4}{r}$$

$$v = \frac{2}{\sqrt{r}}$$

Hence P = mv

$$P = \frac{m2}{\sqrt{r}}$$

20. (b)



using data $\theta = \sin^{-1}\left(\frac{5}{10}\right) = 30°$

$\therefore$ Net force exerted by Rod

$$\Rightarrow 10g\cos30° + \left(10 \times \frac{4\pi^2}{T^2} \times 0.5\right)\cos60°$$

$$\Rightarrow \left(100 \times \frac{\sqrt{3}}{2}\right) + \left(\frac{10 \times 4 \times (3.14)^2}{(1.58)^2} \times \frac{5}{10}\right) \times \frac{1}{2}$$

$$= 85 + \left(10 \times 4 \times 3.94 \times \frac{1}{2}\right) \times \frac{1}{2}$$

$= 85 + (10 \times 3.94)$
$= 85 + 39.4 = 124$ N

21. (d) Since the motion is retarding the body velocity will become zero after time t′

$$O = v - \frac{v^2}{4\pi R} \times t'$$

$t' = \frac{4\pi R}{v}$

which is half of the given time.

$t = \frac{8\pi R}{v}$

Also in t′ the displacement becomes

$S = vt' - \frac{1}{2} a t'^2$

$S = v \times \frac{4\pi R}{v} - \frac{1}{2} \times \frac{v^2}{4\pi R} \times \frac{16\pi^2 R^2}{v^2}$

$S = 2\pi R$ one revolution in rest of time $\left(t'' = \frac{4\pi R}{v}\right)$

it again start from zero velocity to final velocity of v &
therefore complete another revolution.

∴ total no. of revolution Two.

22. (c) As per question
time of drop for falling through height 'H'.

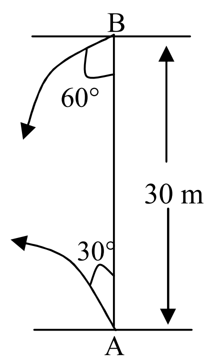
$t = \sqrt{\frac{2H}{g}}$

also $\omega = \frac{2\pi}{t}$ ∴

$\omega = 2\pi \times \sqrt{\frac{g}{2H}}$

$\omega = \pi \times \sqrt{\frac{2g}{H}}$

23. (c) $\text{time} = \frac{\text{distance}}{\text{velocity}}$



$2 = \frac{30}{v}$

v = 15 m/s

as the components of velocities are responsible

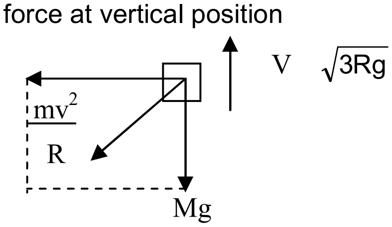
∴ $v_A = 15 \cos 30°$ $\quad$ $v_B = 15 \cos 60°$

$v_A = \frac{15\sqrt{3}}{2}$ $\quad$ $v_B = \frac{15}{2}$

$v_A = 7.5\sqrt{3}$ m/s $\quad$ $v_B = 7.5$ m/s

24. (d) $\theta = 45°$ $\quad$ $\mu = 1$
due to simple fact the μ&θ are same.

25. (a) Net force at vertical position

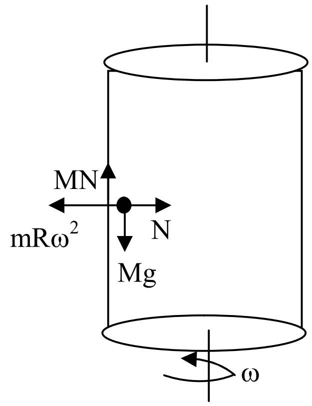


$F = \sqrt{\left(\frac{mv^2}{R}\right)^2 + M^2g^2} = \sqrt{m^2 9g^2 + M^2g^2}$

$F = m\sqrt{10g^2} \Rightarrow F = mg\sqrt{10}$

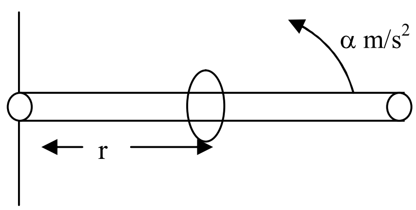
Hence acceleration $\Rightarrow \frac{F}{m} = g\sqrt{10}$

26. (c)



$Mg = \mu N$
$mg = \mu m R \omega^2$
$g = \mu R \omega^2$
or $\mu = \frac{g}{R\omega^2} = \frac{10}{2 \times (5)^2} = \frac{10}{50} = 0.2$

27. (b)



Suppose the slipping starts after time 't'
Centrifugal force = frictional force
$m r \omega^2 = \mu v$

(i) $\omega = \omega_0 + \alpha t$
$\omega = 0 + \alpha t$
$\omega = \alpha t$

(ii) $N = ma = m r \alpha$. (since contact is due to acceleration of rod)

Hence
$\Rightarrow m r (\alpha t)^2 = \mu m r \alpha$
$\Rightarrow \alpha^2 t^2 = \mu\alpha$
$\Rightarrow t^2 = \frac{\mu}{\alpha}$
$\Rightarrow t = \sqrt{\frac{\mu}{\alpha}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

28. (b) From Basic knowledge.
Tension is zero at top point if velocity at Lower most point is $\sqrt{5g\ell}$.

29. (c) As per Question.
$\frac{v^2}{R} = \frac{dv}{dt}$
or $\int_0^t dt = \int_{v_0}^{v} R\frac{dv}{v^2}$
$t = R\left[\frac{1}{v_0} - \frac{1}{v}\right]$ ……(i)

also $\frac{v^2}{R} = v\frac{dv}{dx}$

$$\int_0^{2\pi R} dx = R\int_{v_0}^{v}\frac{dv}{v}$$

$\Rightarrow 2\pi R = R \log_e \frac{v}{v_0}$

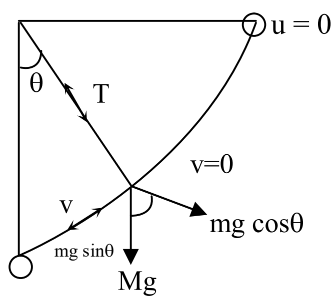
$\Rightarrow e^{2\pi} = \frac{v}{v_0}$ $\Rightarrow v = v_0 e^{2\pi}$ .....(ii)

Putting (ii) in (i)

$$t = R\left[\frac{1}{v_0} - \frac{1}{v_0 e^{2\pi}}\right]$$

$$t = \frac{R}{v_0}\left[1 - e^{-2\pi}\right]$$

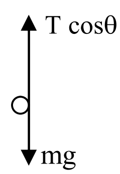
30. (b)



Since the V_y is maximum $\therefore$ ay = 0

Hence $F_y = 0$

By F.B.D.



$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$ ....(i)

By energy conservation.

$v = \sqrt{2g\ell\cos\theta}$ ....(ii)

using (i) & (ii) in $T - mg\cos\theta = \frac{mv^2}{\ell}$

$\Rightarrow \frac{mg}{\cos\theta} - mg\cos\theta = \frac{mv^2}{\ell}$

$\Rightarrow \frac{mg}{\cos\theta} = \frac{m \times 2g\ell\cos\theta}{\ell} + mg\cos\theta$

$\Rightarrow \frac{mg}{\cos\theta} = 3\,mg\cos\theta \Rightarrow \cos^2\theta = \frac{1}{3}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$

31. (b) $Ca^{2+}\,{}^{-}C\equiv C^{-}$, one sigma and two pi bonds.

32. (c) NO has 15 electrons: $KK(\sigma_{1s})^2(\sigma^*_{1s})^2(\pi 2_{p_x})^2(\pi 2_{p_y})^2(\sigma 2_{p_z})^2(\pi^* 2_{p_x})^1$ with bond order 2.5, paramagnetic nature.

NO^+ has 14 electrons, where $(\pi^*_{2p_x})^1$ electron is lost. The bond order increases to 3 and diamagnetic nature is attained.

33. (d) % ionic character = $\frac{\mu obs}{\mu ionic} \times 100$

μ ionic = q × d

$= 4.803 \times 10^{-10} \times 1.275 \times 10^{-8} = 6.12$ D

% ionic character = $\frac{1.03}{6.12} \times 100$

= 16.83

34. (d) Decrease in angle is related with decreases in s-character in hybridisation. Angle 109°28' is related with sp^3 hybridisation, 25%s^- character H_2O has angle 104.5°(<109°28') and so, s-character has to be less than 25% (actual value is 21.43%)

35. (b) Bond order of N_2^{2-} and N_2^{2+} is 2.

Bond order of N_2^- and N_2^+ is 2.5

Bond order of N_2 is 3.

36. (b)

37. (c) $\mu = \sqrt{\frac{3RT}{M}}$, $E = \frac{3}{2}RT \Rightarrow RT = \frac{2}{3}E$

$\mu = \sqrt{\frac{3 \times \frac{2}{3}E}{M}} = \sqrt{\frac{2E}{M}}$

38. (b) log p + log v = constant

Log p = –log v

So slop = –1

39. (c) more is the molecular mass, more will be the partial pressure.

40. (a) Z > 1 means less compressible gas while

Z < 1 means more compressible gas.

41. (a) $P_1 = 100, V_1 = 100, V_2 = 110$

$P_2V_2 = P_1V_1$

$\Rightarrow \quad P_2 = \frac{100 \times 100}{110} = 90.9$

So, decrease in pressure = 100 – 90.9 = 9.1%

42. (b) $\frac{r_{O_2}}{r_{CH_4}} = \frac{n_{O_2}}{n_{CH_4}} = \sqrt{\frac{M_{CH_4}}{M_{O_2}}}$

$= \frac{3}{2} \times \frac{16}{32} \times \sqrt{\frac{16}{32}} = \frac{3}{4\sqrt{2}}$

43. (d) pv = nrt

44. (c) Because the number of moles is constant

$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$; $P_f = \frac{P_iV_iT_f}{V_fT_i}$

$P_f = \frac{P_iV_iT_f}{V_fT_i}$

$= 3.21 \times 10^5 \text{ Pa} \times \frac{V_i}{1.03V_i} \times \frac{(273 + 28.0)}{(273 - 5.00)}$

$= 3.50 \times 10^5$ Pa

45. (b) ke = $\frac{1}{2}mv^2$

46. (a) XeO_3 has 3 bonded electron pairs and two lone pair so have t shape

47. (b) In $\ddot{N}H_3$ central atom is nitrogen which is sp^3 hybridized hence, it will be at the centre of tetrahedron with H-atoms at three vertices.

48. (d)
49. (a)
50. (a) $CO_3^{2-} < CO_2 < CO$
51. (a)
52. (a)
53. (a) PM = dRT

$$\frac{P}{d} = \frac{RT}{M}$$

54. (d) Number of moles of water vapour = $\frac{4.5 \times 10^3}{18} = 250$

Volume of water vapour at STP = $\frac{250 \times 22.4}{1000} m^3$ = 5.6 m^3

55. (b) $\frac{P_1V_1}{T_1}$ (Bottom) = $\frac{P_2V_2}{T_2}$ (Surface)

$$\frac{1.5 \times V_1}{288} = \frac{1 \times V_2}{298}$$

$V_2 = 1.55 \approx 1.6$

56. (d) $$\frac{(\nu_{rms})A}{(\nu_{rms})B} = \frac{\sqrt{\frac{3RT_A}{m_A}}}{\sqrt{\frac{3RT_B}{m_B}}} = \sqrt{\frac{T_A m_B}{T_B m_A}}$$

When $T_A m_B = T_B m_A$, then $(\nu_{rms})_A = (\nu_{rms})_B$.

57. (c) Formal charge = total valence electrons in free atom – number of non-bonding electrons – ½ number of bonding electrons

= 6 – 4 –1/2 × 4 = 0

58. (b) $\mu_{AV(O_2)} = \sqrt{\frac{8RT}{\pi \times 32'}}; \quad \mu_{rms(N_2)} = \sqrt{\frac{3RT}{28}}$

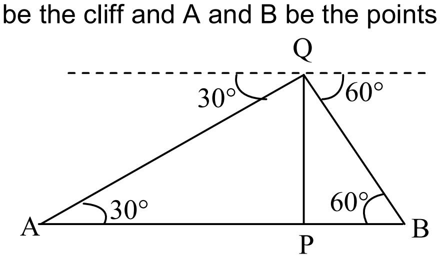
$$\therefore \frac{\mu_{AV(O_2)}}{\mu_{rms(N_2)}} = \sqrt{\frac{8 \times 28}{\pi \times 32 \times 3}} = \sqrt{\frac{7}{3\pi}}$$

59. (d)
60. (c)
61. (a) $$f(x) = \frac{5\sin^3 x \cos x}{\tan^2 x + 1} = \frac{5\sin^3 x \cos x}{\frac{\sin^2 x}{\cos^2 x} + 1}$$

$= 5 (\sin^3 x) (\cos^3 x) = \frac{5}{8} \sin^3 2x$

Hence, maximum value is $\frac{5}{8}$.

62. (b) Let PQ be the cliff and A and B be the points under observation.



$\Rightarrow$ PQ = 100 m, AP = 100 cot 30° = 100 $\sqrt{3}$

and $\quad BP = 100 \cot 60^\circ = \frac{100}{\sqrt{3}}$

$\therefore \quad AB = AP + BP = 100\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$

$= \frac{400}{\sqrt{3}}$ m

63. (a) $\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}} = \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2.2\cos^2 2x}}}}$

$= \frac{2}{\sqrt{2+\sqrt{2+2\cos 2x}}}$

$= \frac{2}{\sqrt{2+2\cos x}}$

$= \frac{2}{2\cos\frac{x}{2}} = \sec\frac{x}{2}$

64. (d) Given equation is $5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0$

$\Rightarrow 5(2\cos^2\theta - 1) + 1 + \cos\theta + 1 = 0$

$\Rightarrow \quad 10\cos^2\theta + \cos\theta - 3 = 0$

$\Rightarrow \quad (2\cos\theta - 1)(5\cos\theta + 3) = 0$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$ or $\cos\theta = -\frac{3}{5}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$ or $\theta = \pi - \cos^{-1}\left(\frac{3}{5}\right)$

65. (d) Given, $\sqrt{3}\sin x + \cos x = 4$

$\Rightarrow \quad 2\left[\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right] = 4$

$\Rightarrow \quad \sin\left(x + \frac{\pi}{6}\right) = 2$ ……………(i)

But $\quad \sin\left(x + \frac{\pi}{6}\right) \le 1$

$\therefore$ No solution exist.

66. (b) $\cos 15^\circ \cos 7\frac{1}{2}^\circ \sin 7\frac{1}{2}^\circ$

$= \frac{1}{2}\cos 15^\circ \sin 15^\circ = \frac{1}{4}\sin 30^\circ$

$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

67. (d) Given that,

$\sin x + \cos x = \min_{a \in R}\{1, a^2 - 4a + 6\}$

Now, $\quad a^2 - 4a + 6 = (a-2)^2 + 2$

$\therefore \quad \min_{a \in R}\{1, a^2 - 4a + 6\} = \min\{1, 2\} = 1$

$\therefore \quad \sin x + \cos x = 1$

$\Rightarrow \quad \sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$\Rightarrow \quad x + \frac{\pi}{4} = n\pi + (-1)^n . \frac{\pi}{4}$

$\Rightarrow \quad x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

68. (a) We have $\left(1+\cos\frac{\pi}{6}\right)\left(1+\cos\frac{\pi}{3}\right)\left(1+\cos\frac{2\pi}{3}\right)\left(1+\cos\frac{7\pi}{6}\right)$

$=\left(1+\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{\sqrt{3}}{2}\right)$

$=\left(1-\frac{3}{4}\right)\left(1-\frac{1}{4}\right) = \frac{1}{4}\times\frac{3}{4}$

$=\frac{3}{16}$

69. (a) Given, $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta$

$= \frac{1}{2}(1-\cos^2\theta) + \frac{1}{3}\cos^2\theta = \frac{1}{2} - \frac{1}{6}\cos^2\theta$

Since, $0 \le \cos^2\theta \le 1$

$\Rightarrow \quad -\frac{1}{6} \le -\frac{1}{6}\cos^2\theta \le 0$

$\Rightarrow \quad \frac{1}{3} \le \frac{1}{2} - \frac{1}{6}\cos^2\theta \le \frac{1}{2}$

$\Rightarrow \quad \frac{1}{3} \le P \le \frac{1}{2}$

70. (c) Since, $-1 \le \cos\theta \le 1$

$\Rightarrow \quad -5 \le 5\cos\theta \le 5$

$\Rightarrow \quad -5 + 12 \le 5\cos\theta + 12 < 5 + 12$

$\Rightarrow \quad 7 \le 5\cos\theta + 12 < 17$

Hence, minimum value is 7.

71. (c) We have $\sin\frac{\pi}{14}\ \sin\frac{3\pi}{14}\ \sin\frac{5\pi}{14}\ \sin\frac{7\pi}{14}\ \sin\frac{9\pi}{14}\ \sin\frac{11\pi}{14}\ \sin\frac{13\pi}{14}$

$= \sin\frac{\pi}{14}\ \sin\frac{3\pi}{14}\ \sin\frac{5\pi}{14} \times 1 \times \sin\left(\pi-\frac{5\pi}{14}\right)\ \sin\left(\pi-\frac{3\pi}{14}\right)\ \sin\left(\pi-\frac{\pi}{14}\right)$

$= \sin^2\frac{\pi}{14}\ \sin^2\frac{3\pi}{14}\ \sin^2\frac{5\pi}{14} = \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2$

$=\left[\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right]^2$

$=\left[-\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right]^2 = \left[\frac{-\sin 2^3\frac{\pi}{7}}{2^3\sin\frac{\pi}{7}}\right]^2 = \frac{1}{64}$

72. (a) We have, $\frac{\tan 3x - \tan 2x}{1+\tan 3x \tan 2x} = 1$

$\Rightarrow \quad \tan(3x - 2x) = 1$

$\Rightarrow \quad \tan x = 1$

$\Rightarrow x = n\pi + \frac{\pi}{4}$

But for this value of x, tan 2x is undefined.

$\therefore$ x $\in \phi$

73. (d) AM $\geq$ GM

$\frac{2\tan A + \sqrt{3}\tan B + \tan C}{3} \geq (2\sqrt{3} \tan A \tan B \tan C)^{1/3}$

$\geq (2\sqrt{3}\ 3\sqrt{3}\ )^{1/3} \geq (18)^{1/3}$

$\Rightarrow \quad 2 \tan A + \sqrt{3} \tan B + \tan C \geq (486)^{1/3}$

74. (b) We have $\tan^2 A \tan^2 B = \frac{1}{3}$

$(2 - \cos 2A)(2 - \cos 2B) = \left(2 - \frac{1-x^2}{1+x^2}\right)\left(2 - \frac{1-y^2}{1+y^2}\right)$

where x = tan A, y = tan B

$= \frac{(1+3x^2)(1+3y^2)}{(1+x^2)(1+y^2)}$

$= \frac{1+9x^2y^2+3(x^2+y^2)}{1+x^2y^2+(x^2+y^2)}$

$= \frac{1+3+3(x^2+y^2)}{1+\frac{1}{3}+(x^2+y^2)} = \frac{3(x^2+y^2)+4}{\frac{3(x^2+y^2)+4}{3}} = 3$

75. (b) We have $1 + \sin\theta + \sin^2\theta \ldots\infty$

$= 4 + 2\sqrt{3}$, $0 < \theta < \pi$, $\theta \neq \frac{\pi}{2}$

$\Rightarrow \frac{1}{1-\sin\theta} = 4 + 2\sqrt{3}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

76. (b) We have $\sin^4 x \leq \sin^2 x$ ……………..(i)

and $\cos^7 x \leq \cos^2 x$ ……………..(ii)

By adding (i) & (ii) we get

$\cos^7 x + \sin^4 x \leq 1$ ………………(*)

But $\cos^7 x + \sin^4 x = 1$ (Given)

and (i), (ii) becomes equality only if $\cos^2 x = \cos^7 x$ and $\sin^2 x = \sin^4 x$ which are satisfied by $x = -\frac{\pi}{2}, \frac{\pi}{2}, 0$.

77. (c) According to given condition $\sin\alpha + \sin\beta = -a$ and $\cos\alpha + \cos\beta = -c$.

$\Rightarrow \tan\frac{\alpha+\beta}{2} = \frac{a}{c}$

Now $\sin(\alpha+\beta) = \frac{2\tan\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}} = \frac{2ac}{a^2+c^2}$.

78. (a) $\sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x}$

$= \sqrt{\sin^4 x + 4(1-\sin^2 x)} - \sqrt{\cos^4 x + 4(1-\cos^2 x)}$

$= \sqrt{(2-\sin^2 x)^2} - \sqrt{(2-\cos^2 x)^2}$

$= (2 - \sin^2 x) - (2 - \cos^2 x) = \cos 2x$.

79. (b) We have $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \tan\left(\frac{\pi}{2}\sin\theta\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \frac{\pi}{2}$ (cosθ + sinθ) = $(2\lambda + 1)\frac{\pi}{2}$, $\lambda \in Z$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{2\lambda+1}{\sqrt{2}}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ (for $\lambda = 0, -1$)

$\theta - \frac{\pi}{4} = 2\lambda\pi \pm \frac{\pi}{4}$, $\lambda \in Z$

$\theta = 2\lambda\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$

$\theta = 2\lambda\pi$ or $\theta = 2\lambda\pi + \frac{\pi}{2}$

But $\theta = 2\lambda\pi + \frac{\pi}{2}$ for $\lambda \in Z$ gives extraneous roots as it does not satisfy the given equation

$\therefore \theta = 2\lambda\pi$ is required solution.

80. (c) Let $a^2\cos 2B + b^2\cos 2A + 2ab\cos(A - B) = X$
and let $-a^2\sin 2B + b^2\sin 2A + 2ab\sin(A - B)$ i (A – B) = Y
$\Rightarrow X - iY = a^2e^{-i2B} + b^2e^{i2A} + 2abe^{i(A-B)}$
$= (be^{iA} + ae^{iB})^2$
$= (b\cos A + bi\sin A + a\cos B - ai\sin B)^2$
$= c^2$.

81. (b) Now, $\cos\theta\cos 2\theta\cos 2^2\theta\ldots\cos 2^{n-1}\theta = \frac{\sin(2^n\theta)}{2^n\sin\theta}$

$= \frac{\sin(\pi-\theta)}{2^n\sin\theta}$ $[\because 2^n\theta = \pi - \theta]$

$= \frac{1}{2^n}$.

82. (d) $1 + \sin x\sin^2 x = 0 \Rightarrow 4 + 2\sin x = \sin 2x$
$\Rightarrow 2 \le \sin 2x \le 6$ which is not possible
$\Rightarrow$ given equation has no solution.
$\therefore$ number of solutions are zero.

83. (a) The given equation is $(\tan 9^\circ + \cot 9^\circ) + (\tan 27^\circ + \cot 27^\circ)$

$= 2\,\text{cosec}\,18^\circ + 2\,\text{cosec}\,54^\circ$

$= 2\left[\frac{1}{\sin 18^\circ} + \frac{1}{\sin 54^\circ}\right]$

$= 2\left[\frac{4}{\sqrt{5}-1} + \frac{4}{\sqrt{5}+1}\right]$

$= 2\left[\frac{4\sqrt{5}+4+4\sqrt{5}-4}{5-1}\right]$

$= \frac{2\cdot 8\sqrt{5}}{4} = 4\sqrt{5}$

84. (a) Standard Formula.

85. (a) $(3\sin\theta + 4\cos\theta)^2 + (4\sin\theta - 3\cos\theta)^2$

$= 9 + 16 = 25$
$\therefore\ 25 + (4 \sin\theta - 3\cos\theta)^2 = 25$
$\Rightarrow 4 \sin\theta - 3\cos\theta = 0$

86. **[a]**

$\cos x = \sqrt{1-\sin 2x}$
$\cos x = |\cos x - \sin x|$
when $\cos x = \cos x - \sin x$
$\Rightarrow \sin x = 0$; $x = 0, \pi$ not possible.
or $\cos x = -\cos x + \sin x$
$\sin x = 2\cos x$
$\tan x = 2$
$x = \tan^{-1} 2$

87. **[c]**

$\sin(x-2) = \sin(3x-4)$
$(x-2) = n\pi + (-1)^n (3x-4)$
Taking $n = 3$ $\quad x = \frac{-3\pi}{4} + \frac{3}{2}$

88.[c] $\because\ \cos^3 A = \frac{3\cos A + \cos 3A}{4}$

Given expression

$= \frac{1}{4}[3\cos x+\cos 3x+3\cos(120-x)+\cos(360-3x)+3\cos(120+x)+\cos(360+3x)]$

$= \frac{1}{4}[3\cos x + 3\cos 3x + 6\cos 120 \cos x]$

$= \frac{3}{4}[\cos 3x]$

maximum value $= \frac{3}{4}$

89.[c]

$\cos^2\frac{\pi}{16} + \cos^2\frac{3\pi}{16} + \cos^2\left(\frac{\pi}{2}-\frac{3\pi}{16}\right) + \cos^2\left(\frac{\pi}{2}-\frac{\pi}{16}\right)$

$\Rightarrow \cos^2\frac{\pi}{16} + \sin^2\frac{\pi}{16} + \cos^2\frac{3\pi}{16} + \sin^2\frac{3\pi}{16}$

$= 1 + 1 = 2$

90. **[b]**

$\sin^{-1}x = \theta + \beta$ and $\sin^{-1} y = \theta - \beta$
$\Rightarrow x = \sin(\theta+\beta)$ and $y = \sin(\theta-\beta)$
put the value and solving we get
$1 + xy = \sin^2\theta + \cos^2\beta$